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# Studying Galileo at Secondary School: A Reconstruction of His 'Jumping-Hill' Experiment and the Process of Discovery

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ABSTRCT. The manuscripts of GALILEO, which were studied more intensively since 1973, offer some insights to his research process. This process differed strangely from the way we find in his publications. Specially we are allowed to guess that experiments played a much more important role for Galileo in spite there is only one manuscript sheet available, which undoubtedly contains measure values. The interpretation of this manuscript (folio 116 v) and the reconstruction of the instrument used (an inclined plane with a horizontal end), allow some very interesting questions (more than answers), which offer also pedagogical applications. The reconstructed experiments with GaIileo's normal inclined plane and with other instruments (pile driver, pendulums) allow further speculations which at least offer very useful "research games" for school classes. Such reconstructions have been made at the Deutsches Museum in Munich, also in contact with teachers and teacher students. The myth of Galileo as Columbus or Prometheus of a new science is at the same time fruitful and hindering for such reflections. This is also true for the so called critical work of historians, who from the 18th century till our presence tried to search for the roots of classica1 physics.

Galileo is more than a famous physicist, more than a hero in the fight for the freedom of scientific thought: he is a myth. We see him as a Prometheus, Ikarus or Columbus in science. Such a view dates already since his life-time, when he was compared by his Aristotelian Catholic enemies with Columbus.[1] This myth of Galileo is one of the important reasons for his popularity in school programs, because myths, I think, are very useful in teaching science. First, there are many cases in today's science, which deal with mythical encounters.[2] Secondly, a "myth" can offer a direct way to the emotions of young students, by using personalization, narrating, and comprehension ( of vast and abstract systematical thinking which would need complicated analytic exploration) in short, a synthetic exploration of a discovery .

Since 1973, we have new opportunities for a direct encounter with Galileo. A large number of manuscripts in Firenze attributed to Galileo's stay in Padua before 1609, have been subjected to a scholarly examination.[3] They can be used in some examples for imitating a personal discussion with this scientist himself. It's not necessary to answer in school the questions important to historians: Was he primarily a scholastic researcher or the first modem experimenter? Was he really heretic?[4] Look at the famous manuscript sheet folio 116 verso (Figure 1) that we are going to focus on. It is in his own handwriting, and we can 'look' into his laboratory and reconstruct his private never published experiments.

This sheet is the only proof we possess that Galileo really made quantita-

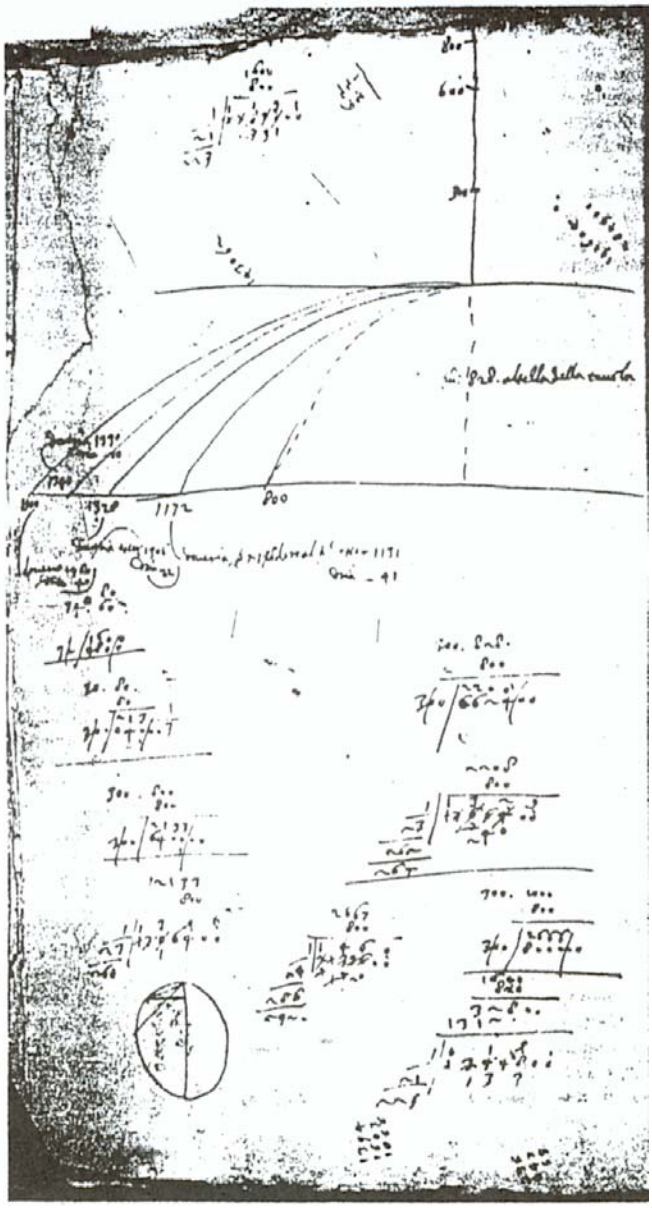

Fig. 1 Manuscript 116 *v* from Galileo, showing his jumping-hill experiments (Biblioteca Nazionale, Florence, Italy).

tive experiments, and that he compared measured values with the calculated ones. This sheet hides some very interesting conceptual problems, which become apparent very surprisingly under a proper scrutiny. And this single sheet allows us to discuss with Galileo the process of discovery. Naturally, we will produce a number of questions and no definitive answers. Thus, it would be desirable to extend the discussion to other manuscripts, letters, and publications. Besides, the historians do not have any further information, which of those writings is *directly* connected to this manuscript.

However, what a teacher can do in school depends on the time he can spare for the topic 'Galileo.' While discussing the historical background and Galileo's scientific biography requires several hours, even one hour devoted to Galileo with a focus on folio 116 v can create a favourable impression on students ( at least, in my experience) .

A teacher can start with any Galileo's purely mythical experiment (e.g. the Pisa-tower experiment) and then to show the manuscript sheet 116 v and a reconstruction of his instrument in the form of a jumping-hill device. Now we will ask (may be, along with the students at school): What can we see in the manuscript and what can be the interpretation? It will become an exciting adventure.

In the original manuscript we see a drawing with a vertical axis carrying the numbers 300, 600, 800 and two horizontal ones. Between these there are 5 curves, starting almost horizontally at a common point. The curves end at the second horizontal axis where we also find numbers (800, 1172, …….).

Everyone who knows a little about Galileo's free fall research, about his inclined -plane experiments, and his description of cannon ball flights in the *Discorsi* will guess: these curves are trajectories of free parabolic flights of bodies (or one body) starting at a common point. Likewise, a modern student even if he is not familiar with Galileo's experiments, but had heard something about free fall in his physics lessons will be able to make a similar guess.

But to justify this guess it is necessary to look at the calculations, mainly at the lower half of the sheet. To understand them better, we will use their English transcript by one historian of science (Figure 2), from which we find at the vertical axis, that the table's height was 828 punti (according to other writings of Galileo 1 punto is equal to 0.94 mm).

Now, if we suppose that the numbers 300, 600, 800, 1000 are the heights of fall in punti, our initial guess about free parabolic flights from the edge of a table (or something similar) becomes more probable!

But how to create a horizontal flight? Quite easily: by means of a jumping-hill device, in which spheres roll down an inclined plane and at its end jump off horizontally. We do not know the exact shape of Galileo's device (how steep it was, and so on), but in principle it had to be a device like the one in Figure 3.

The real proof of our guess comes from the interpretation of the words 'should be. ….. difference. ...' at the end of the curves, coupled with a marked block of calculations. In one of the calculations (on the right side in F), Galileo multiplied 1000 by 800 and then divided it by 300. We see the same pair of values 800 and 300 in other calculations as well. This means it is a reference pair for him. These are the lowest values in the drawing. Then he multiplied the result 2666 by 800 (again) and drew the square root out of it (left side of F). The result is 1460. The whole calculation can be expressed as follows (if we define 1000 = H and 1460 = D):

$$D = \sqrt{\ } [H*800/300]*800$$

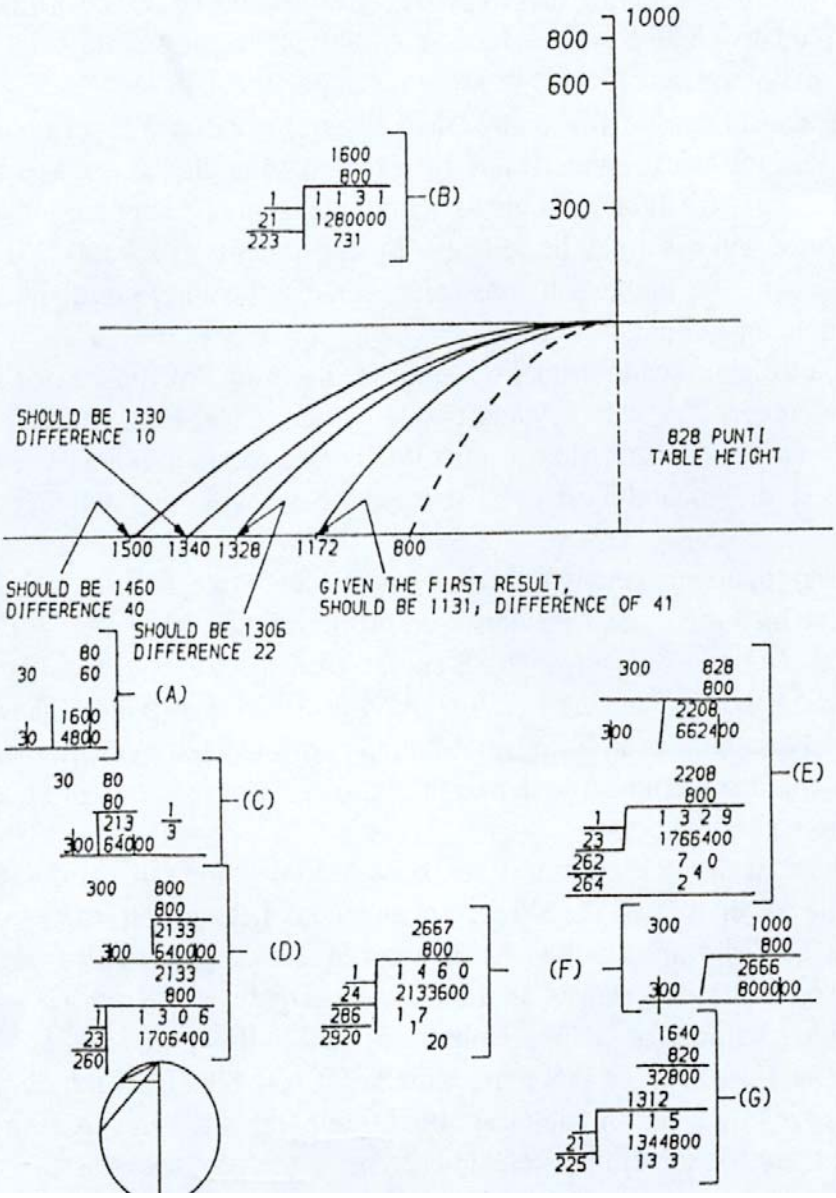


Fig. 2. Transcription of Galileo's manuscript 116 *v* (D.K. Hill, ISIS 77, 1986, p. 287)

Apparently, he was using the relation $D^2 \propto H$ and the first pair of values *(D, H)* to calculate other values of *D* when *H* is given. If Dis the horizontal distance of flight, this confirms our hypothesis very well. As we know, Dis proportional to the horizontal velocity *v* at the common point of all trajectories. But the relation $v^2 \propto H$ is only understandable from a free fall experiment. Indeed, it is equivalent with the free fall law, and Galileo could have shown by two geometrical integrations its equivalence with $s \propto r^2$ .

Before starting to discuss a possible process of Galileo's discovery , that is, what Galileo may have known before this experiment, what he wanted to prove and what remained unclear , I will focus on one of these five curves, which a teacher can use as a pedagogic spotlight. This example displays Galileo's theoretical knowledge, experimental skill, and the

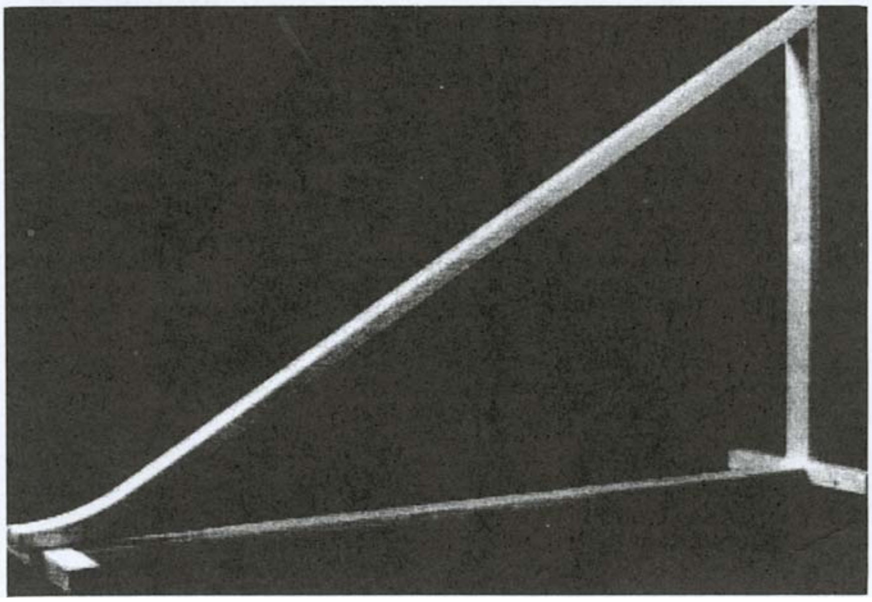

Fig. 3. Galileo's jumping-hill device (Reconstruction Deutsches Museum Munich).

conceptual problems he had to face. In other words, it shows in the nutshell some important parts of the real process of scientific research (it does not matter here that we do not know at all larger context in which this has to be seen).

There are five curves but only four values of height above the table (300, 600, 800, 1000). One value is missing: again this is 828, (see the calculations, Block E).

The curve marked 1340 should correspond to this value. Why did Galileo forget to show this value in the drawing (between the numbers 800 and 1000)? This may be not so important a question (lack of space, etc.). However, there is a much more interesting question: why did he choose the value 828 - exactly the height of the table - for one of the heights, from which the spheres rolled down, while the other heights are rounded to hundreds?

It would be very difficult for students to find an answer to this question. If they know all the mathematical equations for free fall, you can say: Galileo believed that in this case the theoretical value of distance was 2 x 828 = 1656. Let them now solve this problem for the next physics lesson. Historians do not really know whether Galileo believed this or

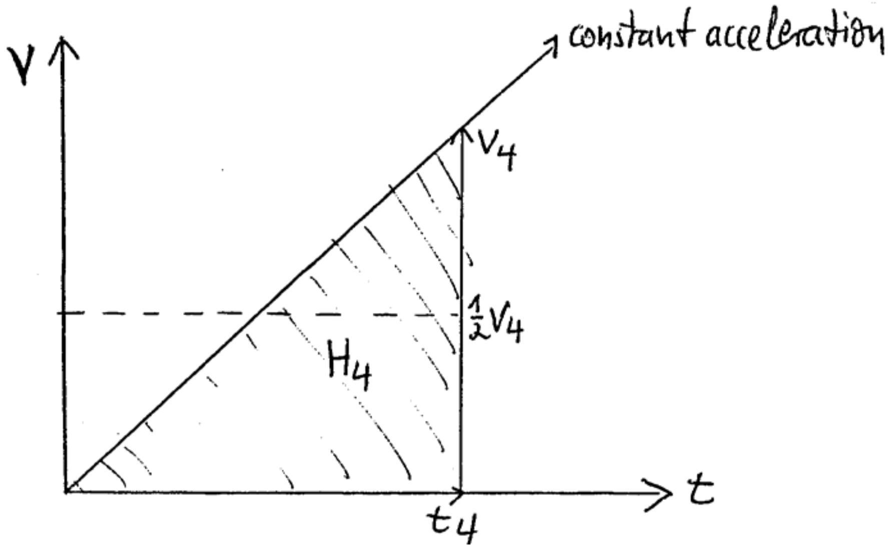


Fig. 4. Geometrical equivalent of Merton regula.

not. But this is probable, and we do not have any other interpretation than that he *expected* this value in an experiment. Galileo used at some points in his free fall research the so called 'Merton rule', known since the time of scholastics. In modern terms, it means that if a body moves with a constant acceleration, and its velocity changes from 0 to *v*, it passes the same distance as if it moved during the same time with the constant velocity *v/2* (Fig. 4).

In our case: if the body would fall free from the height 828 punti and in the time *t4* reaches the velocity $v_4$ (identical with horizontal $v_4$ in the experiment, see figure 5), then, from the free fall law, 828 punti has to be equivalent to 1/2 $v_4$ $t_4$. The parabolic flight - with constant horizontal velocity $v_4$ - now also needs the time $t_4$, because the table's height was 828 punti. But then the expected horizontal distance $D_4$ has to be $v_4t_4$ that is 2 x 828 = 1656.

However, Galileo did not obtain this value, as we see in his manuscript sheet. He received 1340 and compared this value with the one calculated by means of the reference pair 300/800. Perhaps our 1656 is a result of a misinterpretation? Today we know that the value 1656 cannot be a true experimental result because some energy of the moving sphere goes for its rotation, which leaves less energy for the translational motion. This was a conceptual problem, that Galileo couldn't solve.

A modern calculation disregarding friction gives $\sqrt{5/7}$ x 1656 = 1400, which is close to Galileo's experimental value. However, Galileo could not use such an explanation of the discrepancy, for he was unaware of the concept of energy .

But also in his famous experiment in the *Discorsi* conducted with an ordinary inclined plane, Galileo had to see that the ratio $s/t^2$ could not be

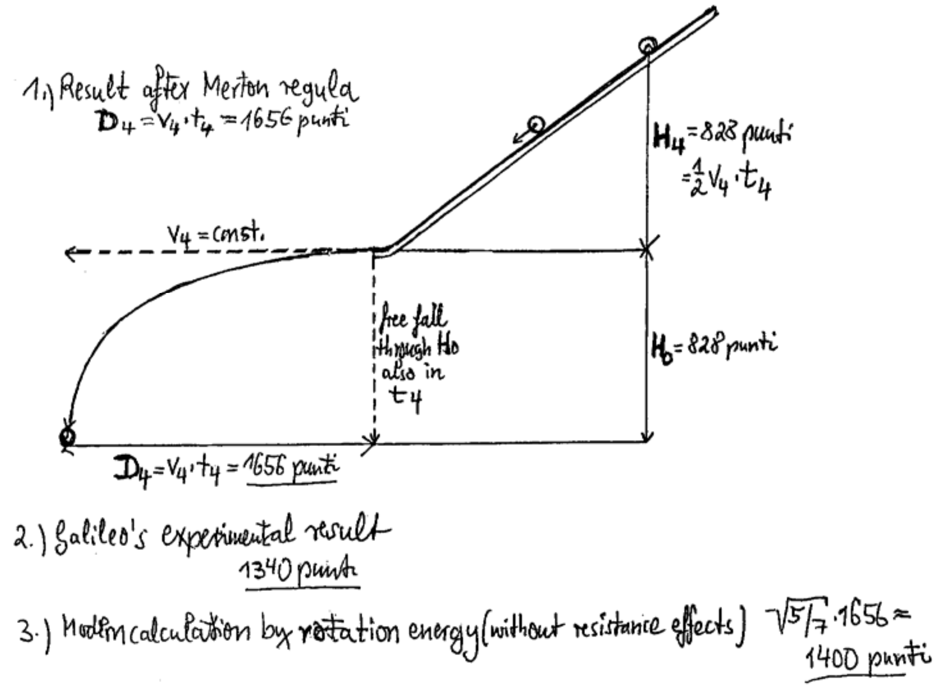


Fig. 5. Galileo's jumping-hill experiment with two equal heights $H_0, H_4$

equal to the ( doubled) value of acceleration of the free fall, which was roughly known from experiments. We do not find *any* comment by Galileo about this discrepancy either in his publications or letters, or manuscripts. The easy explanation of this is that he did the same as many other scientists did after him: when finding some important results they suppressed the small differences which could have delayed the adoption of the new idea if known.

At the time of Galileo, it was customary to present a quantitative law by means of proportions without discussing the constant of proportionality. It is also possible that the comment in question was on another manuscript sheet, which has not reached us. In respect to the absence of such a comment in his publications, we can note that at the time experiments had no importance yet as tools for scientific proofs. Even Galileo's ordinary inclined plane in the *Discorsi* is mentioned only at the end of his long philosophical and geometrical discussion of the concept of constant acceleration and the distance-time relationship. Furthermore, experimental results were always roughly rounded, such as the circumference of the earth's sphere dating from Eratosthenes (see also footnote 6).

Unlike his publications, the manuscript sheet 116 *v* is actually the only proof that Galileo did a careful quantitative experimental work. We have no other explicit example of comparing theoretical and experimental results such precisely from Galileo's times.

Some historians argued that he could have attributed the difference between 1656 and 1340 to the effect of rolling friction-which of course is a total different concept than rotational energy. Were there experiments by which Galileo could have tested this hypothesis of rolling friction (if he wanted it)? In fact, such experiment exists, and there is a very impressive one: using a second jumping-hill device placed opposite to the first one, so that the sphere can rise again from the lowest horizontal part of the first device to the upper part of the second. If the sphere really lost a portion (1340/1656, or about 20% ) of its energy due to friction while descending, it should loose the same amount when going up, and its maximal height can then only be 60% of 828. This is not the case. However, in a real experiment it goes higher. Moreover, the up and down motion of a sphere inside circular concave surfaces also shows less friction then we would expect. Galileo discussed such a motion in the year 1602[8] and compared it with the pendulum motion. One can also find similar remarks in his Discorsi. (Indeed, the pendulum motion could not show to Galileo such problems of rotational energy as the inclined plane) .

Another way of checking the effect of friction can be by changing the angle of inclination of the jumping-hill device. Since experiments show almost no difference, this proves that the 20% loss of expected distance cannot be due to friction.

In respect to this it is helpful to raise the question (perhaps some students will) of why Galileo did not compare rolling spheres to sliding bodies. But if a scientist is not conscious of a specific concept (here, rotational energy), he can stumble on an experiment leading into such a direction only by chance. Moreover, so far as we know the conditions from his sphere experiments, the sliding friction in Galileo's inclined-plane experiments would have been too large. (In his *Discorsi* he used a wooden plank about 6 to 7 m long with a carved groove about 5 cm wide, pasted with parchment to make it smoother). And there is no information that he ever tried to perform the inclined-plane experiments with a very low sliding friction. This would have been a very big effort at the time.

On the other hand, Galileo knew that friction played some role in such experiments (but was not conscious of any concept of rotational energy!). In particular, in the manuscript sheet 116v there is an interesting detail hinting at such a knowledge about frictional resistance. The four distances 1172, 1328, 1340, 1500 measured by Galileo, are all larger than the corresponding numbers 1131, 1306, 1340, 1460 calculated by means of the pair 800/300. When all these distances are obtained with the same jumping-hill device and the same (unknown) angle of incline, this is unexplainable. Because the first height 300 corresponds to the lowest end velocity and shortest running way of the sphere, here the friction has to be less than at the other heights. But that means that all distances beyond the 800 value should have been smaller than the calculated ones (with growing differences). Maybe the first value of 800 came from an experiment with a different angle of incline? This is not impossible, because, unlike other curves, this one is shown by a dashed line.

We do not know anything about such further experiments[9] but in calculation G (Figure 2) Galileo takes now a different reference pair: 820/300. Taking the square root is left unfinished: he gives 115 ..instead of 1160. This would be much nearer to his experimental value 1172 than the former calculation of 1131. And the other values calculated by means of 820/300, are still better! They are slightly larger than the experimental ones. Perhaps he was satisfied with these results and attributed the difference between 820 and 800 to an effect of friction: but this is a speculation.

There are some lesser questions a teacher ( or students) can ask about this sheet 116 *v*. For example, why Galileo used such a complicated way of calculation:

D = *V[H*800/300]*800.*

It seems to be easier to calculate once the constant factor 800 x 300 and then to multiply it by $\sqrt{H}$ . In fact, this is easier only when using an electronic calculator. In any case, it would have been normal *for us* to fix a constant value 800 x 800 in multiplying H. Apparently Galileo uses conventional mathematical forms, in particular proportions, that played an essential role since Greek antiquity.

Sometimes, a question is raised of why Galileo drew the two curves 800 and 1172 in a roughly good proportion to one another but the other curves were "pressed" together. Was this due to lack of space at the sheet? How interesting sometimes even a minor problem seems to be if found in the work of a genius! Another question is: why Galileo used this 'strange' value 828 punti for the 'table height'? This might have been the height of the table in Galileo's laboratory.

One can also ask about the meaning of an isolated figure in the lower left part of the manuscript. Several interpretations have been offered, including the fall along the chords of a circle, and inclined planes with different angles. However, it is also probable that this figure has nothing to do with this manuscript sheet. Because of such isolation of this figure, all these interpretations (with very detailed arguments[10] appear to be highly speculative.

Now, it is the time to ask a question, that may be the most interesting part of a discussion at school (as it proved to be in discussions with teachers at the Deutsches Museum): What were Galileo's goals in the experiment described in the manuscript?

Starting with Stillman Drake (Drake 1973[3]) there has been an animated debate among a few experts, offering different answers to this question. These answers are spotlights on Galileo's process of discovery, and they are also very interesting from a pedagogical point of view:
What is the role of experiments in basic research? How many puzzles

can one experiment resolve at the same time? Indeed, one can uncover in the sheet 116v several very different puzzles:
1) *Was the intention of Galileo to test whether the horizontal distance of a 'jump' was proportional to the horizontal end velocity of the sphere, after rolling down the jumping-hill device?*
The concept of momentanous velocity and its measuring was a big problem for Galileo[11] and his time. It was a philosophical problem: is it possible to have an infinite number of different velocities between two fixed values of it? And it was a mathematical problem as well: how to deal with the instant velocity when neither calculus nor analytical geometry were yet available? See also the discussion of his pile-driver experiments below.
2) *Perhaps he accepted thesis* 1 *for whatever reason and wanted to prove by experiment that the end velocity is proportional to the square root of the height?*
This interpretation, if true, would be of great importance, because $v \propto \sqrt{H}$ is equivalent to the law of free fall $H \propto t^2$ Had he accepted $v \propto t$, Galileo could easily obtain *H* oc f from *v* oc *H.* In the year 1988, the historian D.K. Hill stated that all his colleagues had agreed that this was Galileo's aim.12 But it is alsQ possible that this is only the most interesting solution of this puzzle today rather than the definite one!
3) *Galileo could also have in mind to prove that the horizontal end velocity is conserved, or, in other words , is independent from the vertical motion* . This thesis was supported by S. Drake (1973). We know, that resolving a motion into two components was an interesting methodological question for Galileo, at least, in the direction of its experimental confirmation. The conservation of "movement" was frequently used in his research of pendulum motion.
4) It is possible that he tried to prove that the vertical free fall of a sphere after jumping off is subjected to the same law as its descent along the inclined plane.
5) Or, was his main purpose to discover the geometric form of the flight curve? If so, then perhaps the experiment in question was intended to prove the trajectory of a horizontal free jump to be parabolic. That was the interpretation of R. Naylor.[13]
6) Maybe he also wanted to test the role of friction?
Logically speaking, any of these answers presumes the knowledge of the rest. But a scientist is no philosopher of science. If this experiment was of central importance to Galileo, may be it convinced him directly that all 5 relations are valid! But before making such a conclusion, he had to know something about the basic concepts of motion, such as velocity and acceleration. It is very probable that some of them, e.g. the conservation of a horizontal velocity, were already clear to him at the time of the experiment. However, we know of them only from his later remarks, e.g.

from comparing a pendulum motion to a rolling of a sphere in a semispherical dish.

Interpretations of Galileo's goals in using his jumping-hill device are highly speculative, and it is astonishing that modern historians have tried to be very specific about them! For instance, there were arguments that a simpler experiment, which appears on one of the sheets, should have been executed by Galileo before a more complex one, described on another sheet.[14] I think, the sequence could have been the opposite, for the theoretical context of Galileo's research is unknown to us. Maybe the myth of Galileo is too much alive even in today's historical research. But this myth can therefore help at school to fascinate students, who normally are not interested in the history of science at all.

In any case, the relation $v^2 \propto H$ played an important role, either as a presupposition or as an intermediary hypothesis, or as an experimental result. This is quite clear from the experiment. We know,[15] that Galileo accepted the free fall law $H \propto t$, but believed in the underlying relation $v \propto H$ (instead of $v^2 \propto H$), which could have resulted from experiments with the pile driver (see below). Perhaps, there were also connections to his interest in cannon-ball flights. We find in the *Discorsi* a very detailed discussion of these flights, in particular, of the case of shooting in the horizontal direction.

However, having a few stones left from a building and knowing only its final shape, we cannot reconstruct the sequence of its architect's work.

Now, I will give more details about our pedagogic work in the Deutsches Museum.

My research about this experiment of Galileo was done not only for scientific reasons but mainly for teachers' forthcoming education. We have a hotel inside the museum, where each week a group of teachers receives 5-day courses in the history of science and technology related to their curricula, as well as to actual science and technology.These courses are very successfull. Some teacher groups come from other European countries (Sweden), in which case the course language is English. For those courses I made two different reconstructions of Galileo's jumping-hill device. Like R. Naylor and other historians, I first reconstructed this instrument in such a form as to gain Galileo's maximum height of 1000 punti (940 mm), with the inclination finally fixed to a value of *35°* (see Figure 3).

The experiments with wooden and steel spheres between 15 mm and 40 mm in diameter sometimes brought results very near to those of Galileo (Fig. 6). The success depends on the sphere's weight (a 40 mm steel sphere went the farthest from 940 mm height), on the smoothness of the groove's surface, and on its end being exactly horizontal. That means the friction is really important, as we can also see from our theoretical values (they all are larger than the experimental ones).

For most school purposes, it is more convenient to have a smaller

| *Galileo's Heights* | *Classroom experimental distances* | *Rounded Theoretical Distances* |
|---|---|---|
| 282 | 762 | 790 |
| 564 | 1075 | 1120 |
| 752 | 1233 | 1290 |
| 778 | 1264 | 1320 |
| 940 | 1380 | 1450 |

Fig. 6. Experiments at Deutsches Museum, Munich. Examples of Experimental Results in mm with the Reconstruction of Galileo's Jumping-hill device (angle of declination 35°, height above table 1000 mm, diameter of the groove 50 mm height above ground 778 mm).

reconstruction of this device. The one easy to make is a pipe of metal or plastic at the end of which we fix a flexible stripe of metal, so that the sphere can cross this connection without much friction. When the pipe's inclination is changed, the end of the flexible stripe can stay fixed horizontally. A more elegant device can be obtained by splitting up a plastic pipe, heating one end of it and to bending this end to a certain angle. But in this case the inclination is fixed (see Fig. 7).

This makes sense because slight changes of the angle of inclination do not change the experimental results. But it is interesting to change the height above table, the horizontal exactitude of the end of the device, and the kind of the spheres. To measure the distance of flight, it is possible to fix on the floor a paper strip marked by soot, like it was common in the time of Galileo. We used plastisene, which in addition damped the spheres' impact.

If there is an interest on the part of a teacher to continue the discussion of the role of concepts and experiments in Galileo's works, I would recommend his pile driver experiments. A pile driver was at the time a very important machine and its example is also good for discussing the relations between science and technology. In particular, the teacher can note that Galileo may have learned more about this machine in the famous Arsenale in Venice, a center for shipbuilding.

Galileo described a visit to the Arsenal in the beginning of his *Discorsi*, but his later discussion of the pile driver is not directly related to this visit. That discussion reflects the problem how the effect of the pile drivers' weight increases with the height of its fall. In his publication of 1638 Galileo does not supply a clear answer, but in his earlier correspondence we find the following proportion: The depth to which a post is driven by one stroke, is proportional to the height from which the weight is falling. Naturally, this machine seemed to be a suitable device to measure the instantaneous velocity at the end of the fall (see Fig. 8). Galileo believed that this depth was proportional to the end velocity. Was his conclusion

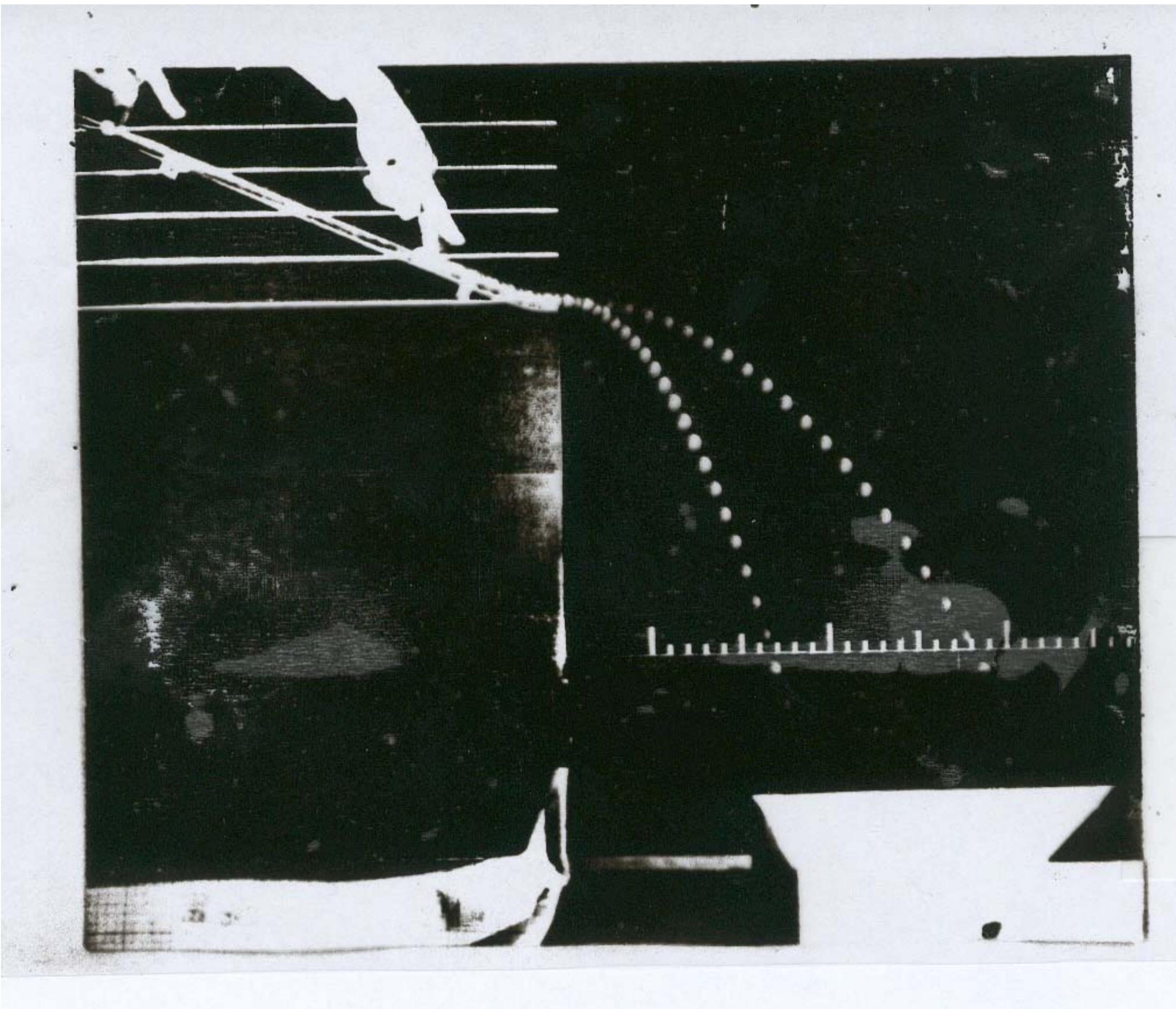

Fig. 7. Smaller model for pedagogic purposes of Galileo's jumping-hill device (Reconstruction Deutsches Museum, Munich).

based on the analogy with the relation between distance and horizontal velocity in his jumping-hill experiment?

This conclusion, Stillman Drake argued, is one important reason why Galileo in the year 16041[5] stated the wrong rule: *the velocity of free fall should be proportional to the distance of the fall.* We do not know if this letter of 1604 is really connected to those experiments. But it is quite clear that Galileo could not have a concept of energy ( a magnitude proportional to the square of velocity). Therefore, he could not choose between two concepts $v$ and $v^2$.

If so, the conclusion from the pile driver experiment had to be that the effect of impact is proportional to the end velocity and the height of fall.

But setting aside these conceptual problems, can we say that experiments hold for the proportionality between the effect of impact and the height of fall?

This is again research in Galileo's laboratory, but more: it's a proof, that machines can directly be used for scientific purposes as our experiments will show.

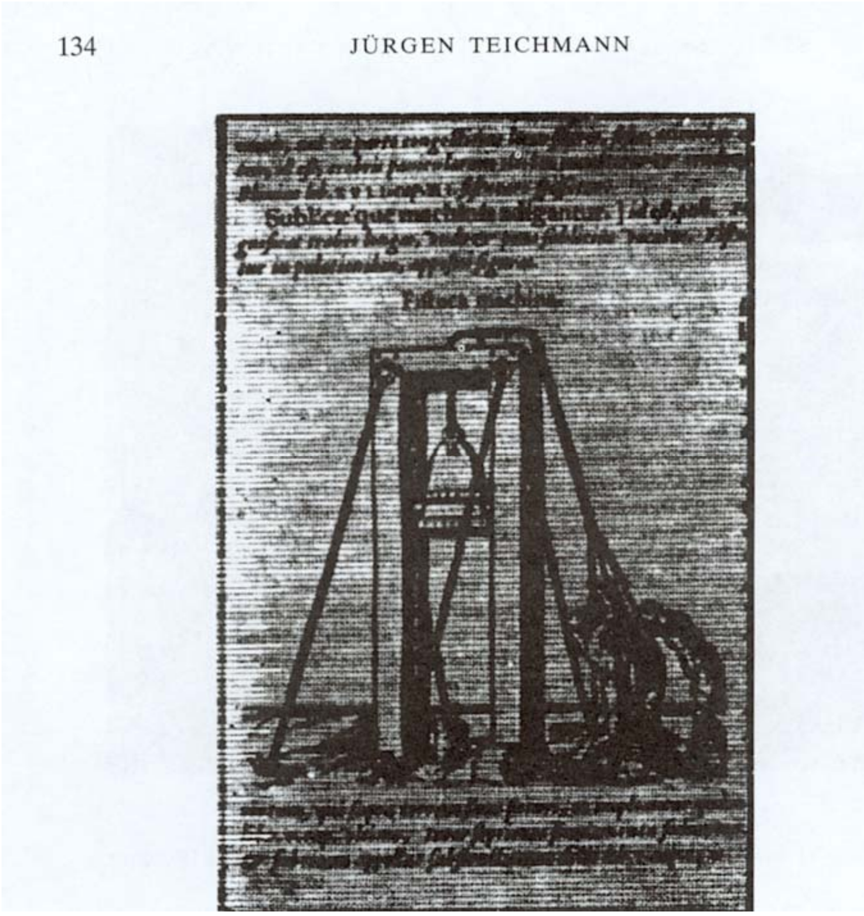


Fig. 8. A historical pile driver .

A classroom reconstruction with small vessels (diameter about 10 cm, with small rods serving as posts) , filled with sand or earth did not show a good proportionality. But experiments performed on a larger scale by teacher students in Munich, which Galileo could have tried in the Arsenal, provided better results.

But the deviations from proportionality are considerable: the precision is much lower than in the jumping-hill experiment. This could be the reason for Galileo to drop his first hypothesis ( effect of impact is proportional to the height of fall) when he found out by simple theoretical arguments *(Discorsi)* that the end velocity cannot be proportional to the height of fall.

## CONCLUSION

I will summarize my experience with both students and teachers as follows: it is a very good, simple, and a relatively short way to show in school how complicated even a "simple" experiment can be. We found that it involved a whole network, from philosophy of science, to conceptual considerations, and to experimental skills. This experiment with Galileo sharpens the view to similarities and differences in more modern science. For instance, teachers supposed that Galileo might have preferred this jumping-hill device because it avoided the difficulty of measuring very short time intervals. Indeed, this is typical for today's science too: for instance, to improve the precision of measuring certain parameters, they are transformed into frequencies, which can be measured very exactly. And students sometimes ask: why did not Galileo discuss his problems at a European meeting of scientists? Such questions may be used to discuss some 'self-evident' features of modern science by comparing them to those of the science of the past.

## ACKNOWLEDGEMENT

My thanks to Naum Kiphnis for help in the English translation of this article.

## NOTES

[1] Redondi, Pietro: 1982, *Galilei der Ketzer.* München, 137.

[2] Hübner, Kurt: 1985, *Die Wahrheit des Mythos.* München.
Bud, Robert: 1993, 'The Museum, Meaning and History: The Case of Chemistry'. In:
*Chemical Sciences in the Modern World,* Philadelphia, pp. 277-294.

[3] See a summary (with much interest in Galileo's experiments) in Damerov, Peter et al.:
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measurement. In: *Proceedings of the American Philosophical Society* , vol. 97, 224.
K. A. 1968: *Metaphysics and Measurement.* London. (See also footnote 1).

[5] Galilei, Galileo, 1638: Discorse e dimostrazioni matematiche .................. Firenze, in H. Crew and
A. De Salvio (eds.), 1950: *Galileo Galilei. Dialogues concerning Two New Sciences.* New York.

[6] In the "Dialogo" 1632 of Galileo we find statements which imply a rate of fall only between
14-15 feet/sec2. But a private note of Galileo (in his own copy of the original edition of the "Dialogo") gives about 32 feet/sec2, which is correct within 3% .
See Naylor, Ronald: 1974, 'Galileo and the Problem of Free Fall', *The British Journal for the History of Science* 7(26), 105-134, here 124.

It's hard to believe that Galileo 1632 really used 14-15 feet1sec2 although he had carried out
such ingenious jumping-hill-experiments many years before.

[7] See R. Naylor citing St. Drake and others in Naylor, 1990 (footnote 3).

[8] Letter of Galileo to Guidobaldo al Monte, 29.11.1602. See: Favaro, Antonio: 1890-1909, *Le Opere di Galileo Galilei.* Edizione Nazionale. 20 vis, Firenze; here vol. 10, 98-100.

[9] The speculations of R. Naylor (Naylor, 1990- see footnote 3) about nonexisting, manuscripts which might have been related to some other inclined plane experiments, are not very helpful.

[10] See Naylor, 1990 (footenote 3), 706. See also the critics by Damerov 1992, 177 , 205 of HiII's interpretations.

[l1] See e.g. Galileo in his "Discorsi" (footnote 5) and St. Drake in Seibold, Eugen and Wolfgang Neuser (eds.), 1990, *Newtons Universum,* Heidelberg.

12 HilI, David K.: 1988, 'Galileo's Early Experiments on Projectile Motion and the Law of Fall', *ISIS* 79, 646-668.

[13] Naylor, R. before 1990. But 1990 Naylor (footnote 3) also concedes that the goals of Galileo have to be more complex than until now discussed.

[14] Naylor, R. before 1990.

[15] Letter of Galileo to Sarpi, 1604. In: *Le Opere* (footnote 8).